diffusion-fundamentals.org

The Open-Access Journal for the Basic Principles of Diffusion Theory, Experiment and Application

Computing the mean square displacement of intruders in freely cooling granular gases with kinetic theory and a random flight approach

E. Abad[1]*, S. B. Yuste[2], V. Garzó[2]

[1]Universidad de Extremadura, Mérida, Spain
[2]Universidad de Extremadura, Badajoz, Spain
*eabad@unex.es

Purpose

We compute the mean square displacement (MSD) of intruders immersed in a freely cooling granular gas composed of inelastic smooth hard spheres ('grains') [1].

Methods

In general, intruders and grains are assumed to have different mechanical properties, implying that non-equipartition of energy must be accounted for in the computation of the diffusion coefficient D. In the hydrodynamic regime, the time decay of the granular temperature T of the cooling gas is known to be dictated by Haff's law [2]; the concomitant decay of the intruder's collision frequency entails a time decrease of D. Explicit knowledge of this time dependence allows us to determine the MSD by integrating the corresponding diffusion equation. To improve our understanding of the observed behaviour, we analyse in detail the properties of the intruder's ballistic displacements inside the granular gas by considering them as random flights.

Results

Previous studies have found a logarithmic time dependence of the MSD in the limit cases when intruders and grains are mechanically equivalent [selfdiffusion] (see e.g. [3,4]) or when the intruder's mass is much larger than that of the grains [Brownian limit] (see [5]). We find that the logarithmic time dependence of the MSD extends beyond the two aforementioned cases, and holds in all spatial dimensions for arbitrary values of the mechanical system parameters (Euclidean spatial dimension d, intruder-grain mass ratio m_0/m, intruder-grain diameter ratio σ_0/σ, and the respective coefficients of normal restitution α_0 and α for intruder-grain collisions and grain-grain collisions). The ultraslow type of diffusion observed here (slower than any power-law in time) is due to its loss of energy in every collision, as opposed to other systems where it arises from the spatial disorder of the medium.

Discussion and Conclusions

For a proper choice of the mechanical system parameters, interesting features emerge from our analysis, such as a non-monotonic dependence of the MSD on the restitution coefficients and on the intruder-hard sphere mass ratio.

References

[1] E. Abad, S.B. Yuste, V. Garzó: *On the mean square displacement of freeling cooling granular gases*, arXiv:2112.06541v1.

[2] V. Garzó: *Granular Gaseous Flows* (Springer, Cham, 2019).

[3] J.J. Brey, M. J. Ruiz-Montero, D. Cubero, R. García-Rojo: *Self-diffusion in freely evolving granular gases*, Phys. Fluids **12**, 876-883 (2000).

[4] A.S. Bodrova, A.V. Chechkin, A.G. Cherstvy, R. Metzler: *Quantifying non-ergodics dynamics of force-free granular gases*, Phys. Chem. Chem. Phys. **17**, 21791-21798 (2015).

[5] J.J. Brey, M.J. Ruiz-Montero, R. García-Rojo, J.W. Dufty: *Brownian motion in a granular gas*, Phys. Rev. E **60**, 7174-7181 (1999).