Diffusion-Reaction in Space-Filling Networks: Oxygen Transport in the Lung

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1. Introduction

The space-filling fractal network in the human lung creates a remarkable distribution system for gas exchange. Landmark studies have demonstrated how the fractal network guarantees minimum energy dissipation [1], slows air down with minimum hardware [2], maximizes the gas-exchange surface area [3], and creates respiratory flexibility between rest and exercise [4]. Here we investigate how the fractal architecture affects oxygen exchange under varying physiological conditions, with respect to performance metrics in terms of diffusive transport, local oxygen currents, and total oxygen current.

We present a renormalization calculation of the diffusion current of oxygen across the branched network of acinar airways, from minimal structural and physicochemical data, which describes how oxygen concentrations drop in the airways as oxygen crosses the alveolar membrane system (stationary diffusion-reaction process) [5]. The calculated oxygen currents agree well with measured values. The results exhibit wide-ranging adaptation to changing process parameters, including the ability to rapidly switch from a low oxygen uptake rate at rest to high rates at exercise, and the ability to maintain a constant oxygen uptake rate in the event of a change in permeability or surface area ("surface poisoning").

2. Optimal performance: quantitative highlights

The possibility of predicting oxygen currents in the lung under varying conditions may give new understanding of the lung's operation, new therapeutic interventions, and new designs for non-biological transport systems. Our calculation, which requires only a pocket calculator, predicts oxygen currents across the lung at four well-defined levels of exercise (rest, moderate exercise, heavy exercise, and "run for your life"), which agree with measured values within a few percent. The calculation treats the acinar airways as a fractal, three-dimensional surface, quantifies the drop in oxygen concentration downstream from the source ("screening") in terms of the average path length an oxygen molecule travels along the membrane before it crosses the membrane ("exploration length," 33 cm under normal conditions), and finds that the network creates (i) a maximum oxygen current across the membrane surface at minimum membrane permeability; (ii) a minimum residence time of oxygen in the airways at maximum exploration of the network; (iii) a maximum current increase when switching from rest to exercise; and (iv) a maximum fault tolerance to loss of permeability or poorly ventilated regions. From an engineering viewpoint, these performance goals may seem mutually exclusive in a single transport architecture. The remarkable finding is that simultaneous realization of these goals is possible and, in the case of the lung, results from an interplay of macroscopic airway structure and microscopic membrane properties.

Specifically, we find maximum fault tolerance to changes in membrane permeability (constant current as a function of permeability) over a permeability range of several decades; near-invariance of trans-membrane oxygen pressure at rest and exercise; and transformation of 180,000 screened gas exchangers at rest into 1,500,000 unscreened exchangers when "running for your life." We show that alternative, less than space-filling architectures perform sub-optimally and that the observed performance of the space-filling architecture results from a competition between underexploration and overexploration of the surface by oxygen molecules.

3. Conclusion

Oxygen exchange across the alveolar membrane system of the lung can be successfully modelled as diffusion-reaction process bounded by a fractal, space-filling surface. The fractal nature of the surface is key to the high performance of the gas exchange system. Operation of the system can be understood in terms of variable degrees of screening under different physiological conditions. Applications to non-biological transport systems, such as porous catalyst systems, will be discussed.

References

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